One Step Equations

An algebraic equation is a mathematical statement that two expressions are equal. A solution of an equation is a value of the variable that makes the equation true. To find the solution, isolate the variable. (make it by itself.)

-Use inverse operations to solve.

Adding/Subtracting

Ex) $y - 8 = 24$ (opp. of -8 is $+8$)

cancels → $+8$ $+8$ ← what we do to one side... we do to the other.

$y = 32$

Ex) $-6 = k - 6$

$+6 \quad +6$

$0 = k$, $k = 0$

Ex) $16 = m - 9$

$+9 \quad +9$

$25 = m$, $m = 25$

Ex) $-\frac{5}{11} + p = -\frac{2}{11}$

$+\frac{5}{11} \quad +\frac{5}{11}$

$p = \frac{3}{11}$

Multiplying/Dividing

Ex) $\frac{y}{3} = -13$

$y = -39$

Ex) $-\frac{8}{3} = j$ (opp of $\div 3$ is $\times 3$)

$-24 = j$, $j = -24$

Ex) $\frac{9y}{9} = \frac{108}{9}$

$y = 12$

Ex) $\frac{5}{6}w = -20 \div \frac{5}{6}$ (KCF)

$-20 \times \frac{6}{5}$

$w = -24$

Solving Multi-Step Equations

Steps:

1. Do I have to distribute?
2. Do I need to clean up? Combine Like Terms
3. Do I have variables on both sides? If so, pick on the little guy.
4. Isolate the variable then add/sub
5. Mult/Divide

Ex) $2(x+5)(x+3) - 3(x-4) = x + 2(2x+4)$

① Distribute? ✓

$2x + 10 - 3x + 12 = x + 4x + 8$

② Clean up? ✓

③ Variables on both sides? ✓

Make the little guy.

$-x + 22 = 5x + 8$

$-2x \quad\quad -2x$

④ Isolate & +or- ✓

$22 = 3x + 8$

$-8 \quad\quad -8$

⑤ Mult/Div. ✓

$\frac{14}{3} = \frac{3x}{3}$

$x = 7$

Ex) $8x - 21 - 5x = -15$

$3x - 21 = -15$

$+21 \quad\quad +21$

$\frac{3x}{3} = \frac{6}{3}$

$x = 2$

① Distribute? X No

② Clean up? ✓

③ Variables on both sides? X No

④ Isolate & + or - ✓

⑤ Mult/Div. ✓

understood (-1)

Ex) $10y - (4y + 8) = -20$

$10y - 4y - 8 = -20$

$+8 \quad +8$

$\frac{6y}{6} = \frac{-12}{6}$

$y = -2$

① Distribute? ✓
② Clean Up? ✓
③ Variables on both sides? X No
④ Isolate + or - ✓
⑤ Mult/Div ✓

Ex) $x + 2(2x+3) - 1 = \frac{1}{2}(4x+28)$

$x + 4x + 6 - 1 = 2x + 14$

$5x + 5 = 2x + 14$

$-2x \quad -2x$

$\frac{3x}{3} = \frac{9}{3}$

$x = 3$

① Distribute? ✓
② Clean Up? ✓
③ Variables on both sides? ✓
④ Isolate + Add/Sub ✓
⑤ Mult/Div ✓

Literal Equations

What is a literal equation?
a literal equation is an equation w/ more than one variable.

Steps to solve:
1. Determine the variable you are solving for.
2. Move all other terms & variables to the other side by isolating the variable & using inverse operations.
3. Rewrite the equation with the isolated variable

Solving Algebra Literal Equations -

- Simple Interest $I = Prt$
  - Solve for t.

$$\frac{I}{Pr} = \frac{Prt}{Pr}$$ ← Isolate

Being MULT. so we divide.

$$t = \frac{I}{Pr}$$ Rewrite

- Slope-Intercept Form $y = mx + b$
  - Solve for m.

$$y - b = mx$$

$$\frac{y-b}{x} = \frac{mx}{x}$$

$$m = \frac{y-b}{x}$$

• Standard Form of a Linear Equation $Ax + By = C$

- Solve for y.

$$Ax + By = C$$
$$-Ax \qquad -Ax$$
$$\frac{By}{B} = \frac{C - Ax}{B}$$
$$y = \frac{C - Ax}{B}$$

Solving & Graphing Inequalities

*same a solving equations, BUT whenever you multiply or divide by a negative you flip the sign.

Steps:

1. Get all variables to one side of the inequality.
2. Solve the inequality and write in order (variable, sign, #)
3. Make a number line
4. Open or closed circle?
5. Shade w/ arrow in the appropriate direction.

Open Circle	Closed circle
$>$ $<$	$\geq$ $\leq$
(No equal bar)	(equal to bar)
Shade Right	**Shade Left**
$>$ $\geq$	$<$ $\leq$
(greater)	(less)

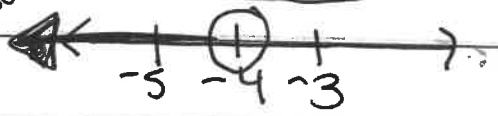
Ex) $-5x + 10 > 30$ (last)

$-10 \quad -10$

$\frac{-5x}{-5} > \frac{20}{-5}$ ← divided by a (−) flip sign

$x < -4$

No equal bar (open circle) less so shade left.

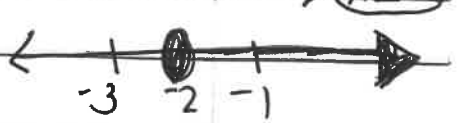


Ex) $5 - 3x \leq 13 + x$

$-13 \quad +3x \quad -13 + 3x$

$\frac{-8}{4} \leq \frac{4x}{4}$

$-2 \leq x$ → $x \geq -2$



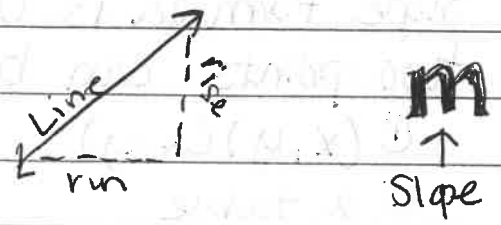
★Inequality Phrases

"at least" $\geq$ greater or equal
"No more than" $\leq$ less or equal
"at most" $\leq$ less or equal

What is Slope?

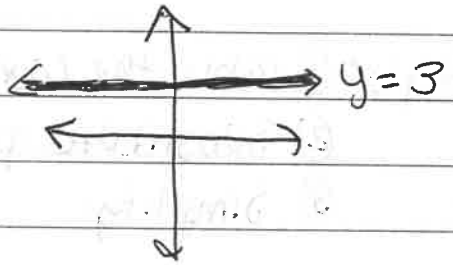
$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

"rate of change"

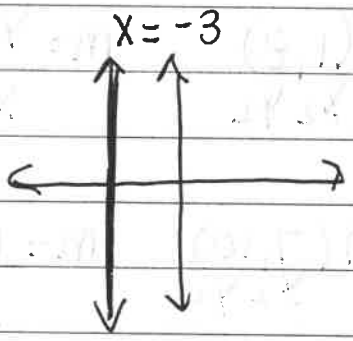


- Slope of vertical & horizontal lines

H	O	V
Horizontal Line	zero Slope	y-axis



V	Vertical Line
U	Undefined Slope
X	X-axis



Slope-Intercept Form

$$y = mx + b$$

m ↓ Slope b ↓ y-intercept

Slope Formula

Slope formula is used when given two points. two points can be given as the following:

① $(x,y)(x,y)$
② a table
③ a graph

Ex 1) Given a line that passes through $(3,-6)$ & $(1,8)$

Steps: ① label the coordinates $(x_1,y_1)(x_2,y_2)$
② Substitute points into the formula $m=\frac{y_2-y_1}{x_2-x_1}$
③ Simplify

$(3,-6)(1,8)$
$x_1\ y_1\ x_2\ y_2$

$m=\frac{y_2-y_1}{x_2-x_1}=\frac{8+6}{1-3}=\frac{14}{-2}$ $\boxed{m=-7}$

Ex 2) $(4,7)(7,10)$
$x_1\ y_1\ x_2\ y_2$

$m=\frac{10-7}{7-4}=\frac{3}{3}$ $\boxed{m=1}$

Ex 3)

x	y
0	3
2	11
4	19
6	27
8	35

(+2, +8 each step)

* Pick any two points

$(2,11)(8,35)$
$x_1\ y_1\ x_2\ y_2$

$m=\frac{35-11}{8-2}=\frac{24}{6}=$ $\boxed{m=4}$

$m=\frac{8}{2}=4$

Linear Forms

Solving for Y-Intercept

① Slope-Intercept $y = \underset{\text{slope}}{m}x + \underset{\text{y-int}}{b}$

We need slope & y-intercept

Ex) given a slope of $-\frac{1}{4}$ & y-int of 3 write an equation. $y = -\frac{1}{4}x + 3$

② Point-Slope Form $y - y_1 = m(x - x_1)$

we need slope & a point. (m: slope; (x_1, y_1): coordinate point)

Ex) Write the equation in point-slope form & solve for y given $m = -\frac{1}{3}$ & $(-1, -6)$. (x_1, y_1)

$y + 6 = -\frac{1}{3}(x + 1)$

$-\frac{1}{3}x - \frac{1}{3}$

-6

$y = -\frac{1}{3}x - \frac{19}{3}$

Ex) Given $m = 3$ & $(-2, 5)$

$y - 5 = 3(x + 2)$

$3x + 6 + 5$

$y = 3x + 11$

What is the only thing you can find from two pts?

Ex) Given $(2, -3)$ $(4, -2)$ (x_1, y_1) (x_2, y_2)

$\frac{-2+3}{4-2} = m = \frac{1}{2}$ → Now use slope & a pt

$y + 3 = \frac{1}{2}(x - 2)$

$\frac{1}{2}x - 1 - 3$

$y = \frac{1}{2}x - 4$

(3) Standard Form $Ax + By = C$

In order to solve for slope-intercept form you must isolate the "y".

$$\underset{-Ax}{Ax} + B\boxed{y} = \underset{-Ax}{C} \quad \text{(last)}$$

$$\frac{By}{B} = \frac{-Ax}{B} + \frac{C}{B}$$ > simplify in slope-int form in the correct order

Ex) $\underset{-5x}{5x} - 2\boxed{y} = \underset{-5x}{10}$ (last)

$$\frac{-2y}{-2} = \frac{+5x}{+2} + \frac{10}{-2}$$

$$y = \frac{5}{2}x - 5$$

Ex) $\underset{+3x}{-3x} + 5\boxed{y} = \underset{+3x}{30}$ (last)

$$\frac{5y}{5} = \frac{3x}{5} + \frac{30}{5}$$

$$y = \frac{3}{5}x + 6$$

Graphing Linear Equations

A Linear Equation makes a straight line on a coordinate plane. & is simplified to $y=mx+b$

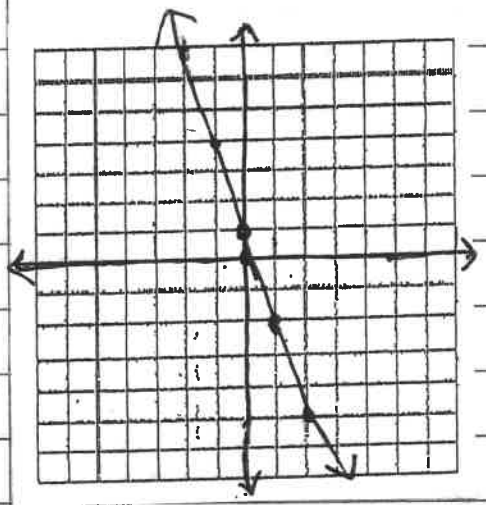
4 Simple Steps to graph:

1. Make sure the equation is in slope-intercept form. (if not solve for y) $y=mx+b$
2. Plot your first point on the y-axis using the y-intercept
3. Use rise over run to graph the next point. & start from the y-intercept.
4. Connect points to form a straight line.

Ex 1: $y=-3x+1$

$m=\frac{-3}{1}$

$b=1$

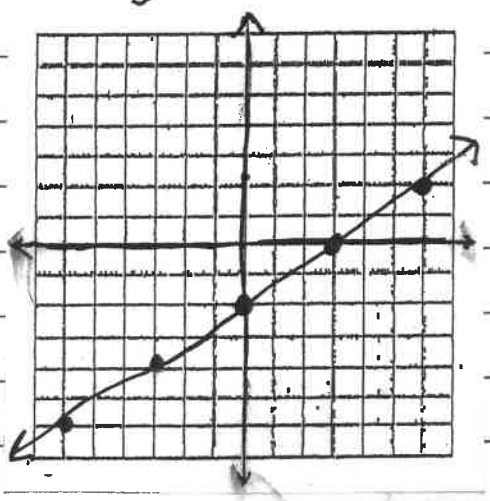


Ex 2: $\frac{2x}{-2x}-\frac{3y}{}=\frac{6}{-2x}$ Solve for "y"

$\frac{-3y}{-3}=\frac{12x+6}{-3}$

$y=\frac{2}{3}x-2$

$m=\frac{+2}{3}$ $b=-2$



Linear Inequalities

a Linear Inequality is an inequality in the form of slope-intercept form, but instead of an equal sign, an inequality symbol is used.
- also many values are part of the solution to linear inequalities.

Steps:
1. Make sure the inequality is in slope-intercept form.
2. Graph the y-intercept on the coordinate plane.
3. Use the slope to "rise over run" to find the next point (starting from the y-intercept).
4. Connect points w/ a solid or a dashed line.
5. Shade above or below.

Solid Line	Dashed Line
$\geq$ $\leq$	$>$ $<$
"equal bar"	"No equal bar"
Shade Above	**Shade Below**
$>$ $\geq$	$<$ $\leq$
Greater	Less.

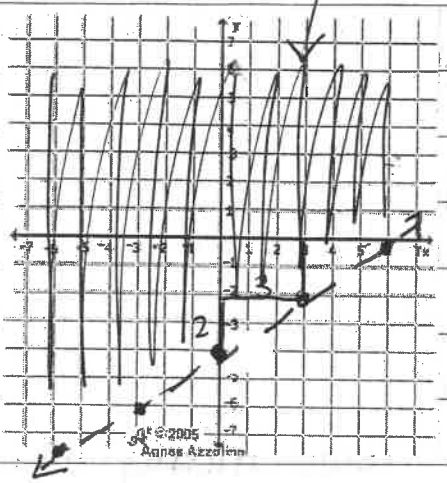
Solutions in the shaded area.

dashed line
shade above

Ex 1) $y \geq \frac{2}{3}x - 4$

$m = \frac{2}{3}$

$b = -4$



Ex 2) $-4x + 2y \geq 10$

$+4x \qquad +4x$

$\frac{2y}{2} \geq \frac{4x}{2} + \frac{10}{2}$

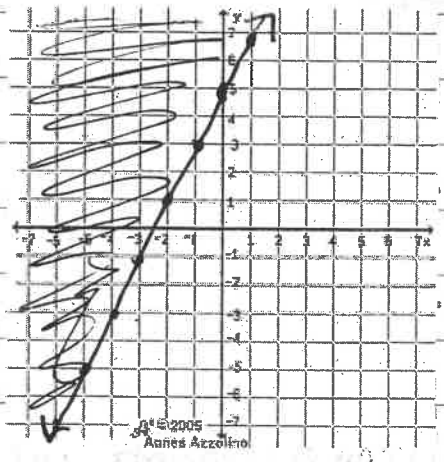
solid line

$y \geq 2x + 5$

shade above

$m = \frac{2}{1}$

$b = 5$



Systems of Equations - Graphing

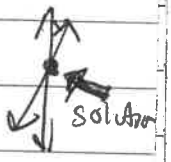
- Systems of equations is where two or more equations that may share a common point (or many points) (or no points)

Steps to Graph Systems:

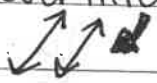
1. Make sure all equations are in slope-intercept form. ($y=mx+b$)
2. Graph each equation.
3. Identify the solution.

3 Types of Solutions:

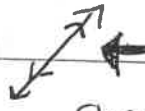
1. One solution (x, y) lines that intersect



2. No Solutions ($\emptyset$) parallel lines never intersect (same slope)

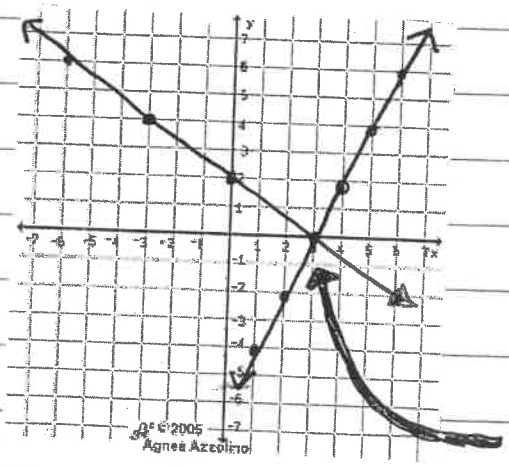


3. Infinitely Many Solutions (IMS) Lines coincide with each other (meaning Identical)



Ex 1)

$$\begin{cases} y = \frac{-2}{3}x + 2 \\ y = 2x - 6 \end{cases}$$



Solution: (3, 0)

Ex 2)

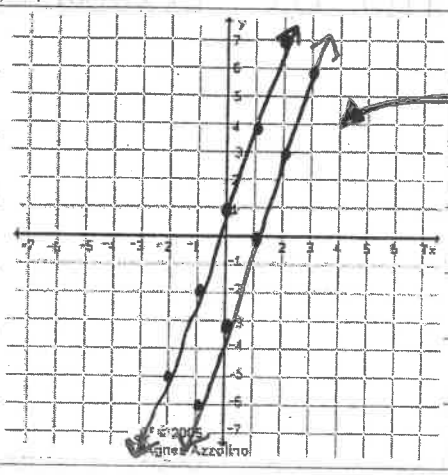
$\{$ $y - 1 = 3x$

✓C $y = 3x - 3$

$y - 1 = 3x$
$+1 \quad +1$

✓ $y = 3x + 1$

* same slopes



Never intersect

$\emptyset$

Solution: $\emptyset$

Ex 3)

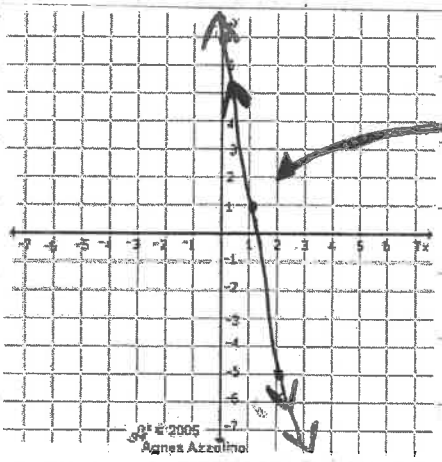
✓$\{$ $y = -6x + 7$

C $12x + 2y = 14$
$-12x \quad -12x$

$\frac{2y}{2} = \frac{-12x}{2} + \frac{14}{2}$

✓ $y = -6x + 7$

Coinciding (identical) lines



Shares every point

Solution: IMS

Systems of Equations - Substitution

Steps:

1. Solve one equation for a variable ($x=$ or $y=$).
2. Substitute the expression into the other equation & solve.
3. Substitute the value found in step two into one of the original equations and solve.
4. Identify the solution.

3 Types of Solutions:

1. One solution (x, y) - lines that intersect.
2. NO solutions $\emptyset$ - parallel lines a lines that never intersect (same slope).
3. Infinitely Many Solutions (IMS) - lines are identical (coinciding)

Ex1) $\begin{cases} y = 2x - 6 \\ 2x + 3y = 6 \end{cases}$ $(3, 0)$ x y

$2x + 3(2x - 6) = 6$

$2x + 6x - 18 = 6$

$8x - 18 = 6$

$+18 \quad +18$

$\frac{8x}{8} = \frac{24}{8}$

$x = 3$

$y = 2x - 6$

$y = 2(3) - 6$

$6 - 6$

$y = 0$

Ex 2) $\begin{cases} 2x+8y=1 \\ x=-4-4y \end{cases}$ $\emptyset$

$2(-4-4y)+8y=1$

$-8-8y+8y=1$

$-8=1$

✗ False Statement

Ex 3) $\begin{cases} 4x-5y=-2 \\ x-2y=4 \\ +2y \quad +2y \\ x=2y+4 \end{cases}$ $(-8, -6)$ x, y

$4(2y+4)-5y=-2$

$8y+16-5y=-2$

$3y+16=-2$

$-16 \quad -16$

$\frac{3y}{3}=\frac{-18}{3}$

$y=-6$

$x=2y+4$

$x=2(-6)+4$

$-12+4$

$x=-8$

Systems of Equations - Elimination

Steps:

1. Both equations must be in Standard Form ($Ax + By = C$)
2. Choose a variable to eliminate and find the LCM (if needed) in order to eliminate & solve.
3. Find the remaining variable by substituting the value into an original equation.
4. Identify the solution.

3 Types of Solutions

1. one solution (x, y) - lines that intersect
2. No Solutions $\emptyset$ - Parallel lines are lines that never intersect (same slope)
3. Infinitely Many Solutions (IMS) - lines are identical (coinciding).

Ex 1) $\begin{cases} 4x - 6y = -14 \\ -4x + y = -11 \end{cases}$

$-5y = -25$

$\frac{-5y}{-5} = \frac{-25}{-5}$

$y = 5$

$(4, 5)$
x, y

$4x - 6(5) = -14$

$4x - 30 = -14$

$+30 \quad +30$

$\frac{4x}{4} = \frac{16}{4}$

$x = 4$

Ex 2) $\begin{cases} 2x+3y=6 \\ 3(2y-y=6) \end{cases}$ $(3,0)$ x, y

$6y-3y=18$

$2y+3y=6$

$\frac{8y}{8}=\frac{24}{8}$

$y=3$

$2(3)+3y=6$

$6+3y=6$

$-6 \quad -6$

$\frac{3y}{3}=\frac{0}{3}$

$y=0$

Ex 3) $\begin{cases} 3(-4x+9y=-28) \\ -4(3x-2y=2) \end{cases}$ $(-2,-4)$ x, y

$-12x+27y=-84$

$+12x-8y=8$

$\frac{19y}{19}=\frac{-76}{19}$

$y=-4$

$3x-2y=2$

$3x-2(-4)=2$

$3x+8=2$

$-8 \quad -8$

$\frac{3x}{3}=\frac{-6}{3}$

$x=-2$

Relations & Functions

Relation: a set of ordered pairs: (x, y)

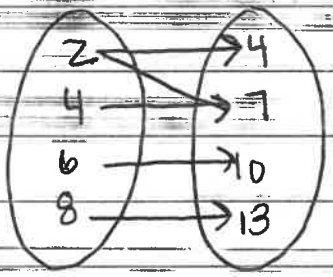
Domain: the x-values of a relation (or input)

Range: the y-values of a relation (or output)

Function: a special type of relation where every x-value has exactly one y-value (x's can't repeat)

Ordered Pair: {(2, 4), (4, 7), (6, 10), (8, 13), (2, 7)}

Mapping Diagram



Tables

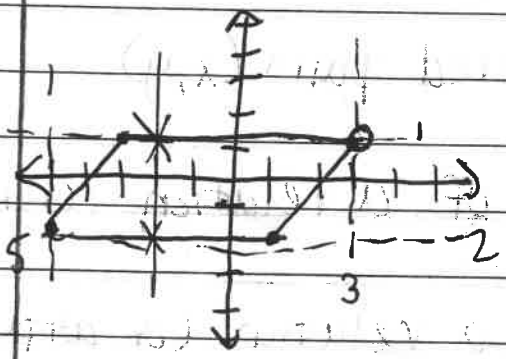
x	y
2	4
4	7
6	10
8	13
2	7

Domain: 2, 4, 6, 8

Range: 4, 7, 10, 13

Function: No because the 2's repeat

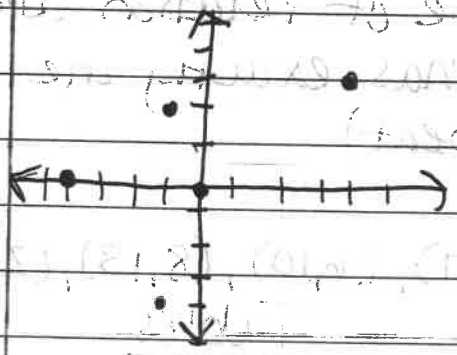
Domain, Range & Function on a Graph



D: $-5 \leq x < 3$ or $[-5, 3)$

R: $-2 \leq y \leq 1$ $[-2, 1]$

F: No



$(-4, 0)(-1, 2)(-1, -5)(0, 0)(4, 3)$

D: $-4, -1, 0, 4$

R: $0, 2, -5, 0, 3$

F: No

Evaluating Functions

We can evaluate functions by plugging in the input values (x-values) to get our output values (y-values).

Ex) $y = -2x + 9$ $x = -3$

$x = -4$ $y = -2(3) + 9$

$f(x) = 7$ $y = 6 + 9$

$y = 3$

$y = -2(-4) + 9$

$y = 8 + 9$

$y = 17$

$7 = -2x + 9$

$-9 \quad -9$

$\frac{-2}{-2} = \frac{-2x}{-2}$ $x = 1$

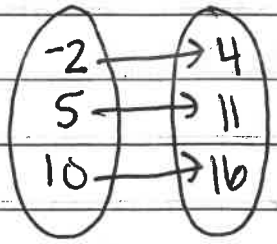
What does f(x) represent.

f(x) is a function notation which is a fancy way to say "y"

f(x) can also be written as any variable, g(x), h(x), f(n), etc.

The variable in the parenthese is the input.

Ex) Complete the Mapping Diagram



$f(x) = x + 6$

$f(-2) = -2 + 6 = 4$

$f(5) = 5 + 6 = 11$

$f(10) = 10 + 6 = 16$

Ex) Find the missing value of the ordered pair.

$$f(x) = 3x + 1$$

$\{(0, ?)\ (?, 7)\ (?, 13)\}$

$(0, 1)$ $(2, 7)$ $(4, 13)$

$f(0) = 3(0) + 1 = 1$

$7 = 3x + 1$

$-1 \quad -1$

$\frac{6}{3} = \frac{3x}{3}$ $\quad x = 2$

$13 = 3x + 1$

$-1 \quad -1$

$\frac{12}{3} = \frac{3x}{3}$ $\quad x = 4$

Writing Functions

Writing functions in linear form is simple.

What two things do we need to write a function? ① slope
② y-intercept.

Linear Functions are written in the form $y = mx + b$.

Ex) Caleb has already spoken on the phone this month for 150 minutes, he continues to speak on the phone for 75 minutes per day. Write a function to model the situation. How many minutes has he talked after 5, 8, 13 days? $f(d) = 75d + 150$

$75(5) + 150 = 525$ min

$75(8) + 150 = 750$ min

$75(13) + 150 = 1,125$ min

Ex) The phone company charges a flat fee of $60 and $5/GB of data. Write a function to model the situation. How much money is spent after 12 GB of data.

$f(x) = 5x + 60$

$f(12) = 5(12) + 60$

$f(12) = 120$

Ex) Write a function given the table.

x	y
1	6
2	7
3	8
4	9

(1) Slope Formula

$\frac{7-6}{2-1} = 1$

(2) Point-Slope

$y - 6 = 1(x-1)$

$y - 6 = 1x - 1$

$+6 \quad +6$

$y = 1x + 5$

Ex) Write a function given the table.

x	y
4	2
7	5
10	8
30	28

$\frac{5-2}{7-4} = \frac{3}{3} = 1$

$y - 2 = 1(x-4)$

$+2 \quad 1x - 4 + 2$

$y = 1x - 2$

Ex) ~~Write a function~~ given the table.

x	y
16	8
24	16
28	20
60	52

$\frac{16-8}{24-16} = \frac{8}{8} = 1$

$y - 8 = 1(x-16)$

$+8 \quad 1x - 16$

$+8$

$y = 1x - 8$

Arithmetic Sequence

An arithmetic sequence is a sequence that forms a pattern by the same nonzero number by adding or subtracting.

An arithmetic sequence forms a linear function b/c of the constant rate of change or common difference.

$$a_n = a_1 + (n-1)(d)$$

D: common difference
N: term we are finding
a_1: 1st Term

Ex) 9, 13, 17, 21, ..., 25, 29, 33
(+4, +4, +4) $d = 4$

Ex) $-\frac{3}{4}, -\frac{1}{4}, \frac{1}{4}, \frac{3}{4}, \ldots, \frac{5}{4}, \frac{7}{4}, \frac{9}{4}$
$d = \frac{1}{2}$

Ex) The 25th term; $a_1 = -5$, $d = -2$

$a_{25} = -5 + (25-1)(-2)$

$a_{25} = -53$

Ex) Find the 16th term: 4, 8, 12, 16, ...

+4 +4 +4

$d = 4$

$n = 16$

$a_1 = 4$

$a_{16} = 4 + (16-1)(4)$

$a_{16} = 64$

Ex) Find the 60th term: 11, 5, -1, -7

-6 -6 -6

$d = -6$

$n = 60$

$a_1 = 11$

$a_{60} = 11 + (60-1)(-6)$

$a_{60} = -343$

Ex) Find the "nth" term: 13, 23, 33, 43, ...

+10 +10 +10

$d = 10$

$n = n$

$a_1 = 13$

$a_n = 13 + (n-1)(10)$

$13 + 10n - 10$

$a_n = 3 + 10n$

Ex) Find the "nth" term: -31, -34, -37, -40, ...

-3 -3 -3

$d = -3$

$n = n$

$a_1 = -31$

$a_n = -31 + (n-1)(-3)$

$-31 + -3n + 3$

$a_n = -3n - 28$